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Energy Distribution Analysis of Impact Signals Based on Wavelet Decompositions

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May 2006

Interim Report for April 2004 to September 2004

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REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. **PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE		3. DATES COVERED (From - To)
May 2006	Interim		Apr 06 – Sep 06

4. TITLE AND SUBTITLE

Energy Distribution Analysis of Impact Signals Based on Wavelet Decomposition

5a. CONTRACT NUMBER
FA8650-04-D-6472

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
62202F

6. AUTHOR(S)

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5d. PROJECT NUMBER
7184

5e. TASK NUMBER
02

5f. WORK UNIT NUMBER
11

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) AND ADDRESS(ES)
Air Force Materiel Command
Air Force Research Laboratory, Human Effectiveness Directorate
Biosciences and Protection Division
Biomechanics Branch
Wright-Patterson AFB OH 45433-7947

8. PERFORMING ORGANIZATION REPORT NUMBER

AFRL-HE-WP-TP-2006-0077

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

AFRL/HEPA

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited. Cleared by AFRL-WS-04-1445, 12/20/04.

13. SUPPLEMENTARY NOTES
Authors' final manuscript, Proceedings of the 75th Shock and Vibration Symposium, Oct 04

14. ABSTRACT

Impact signals are decomposed on orthogonal wavelet bases or wavelet packet bases. Based on these orthogonal decompositions, the energy distribution of an impact signal can be defined in two ways: (a) with respect to natural order of nodes or frequency index, representing the energy distribution at each node or frequency index; (b) with respect to time position and frequency index, providing an energy map over the time-frequency plane. Thus defined energy distributions of impact responses can be used for the validation of finite element automobile crashworthiness modeling by comparing energy distributions between the signals from tests and simulations.

15. SUBJECT TERMS
Orthogonal decompositions, wavelet, energy distribution

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Nathan Wright
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (include area code)
U	U	U	SAR	12	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. 239.18

Energy Distribution Analysis of Impact Signals Based on Wavelet Decompositions

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Impact signals are decomposed on orthogonal wavelet bases or wavelet packet bases. Based on these orthogonal decompositions, the energy distribution of an impact signal can be defined in two ways: (a) with respect to the natural order of nodes or the frequency index, representing the energy distribution at each node or frequency index; (b) with respect to the time position and the frequency index, providing an energy map over a time-frequency plane. By comparing the energy distributions between the signals from tests and simulations, finite element impact models can be validated. A finite element automobile crashworthiness model is used to illustrate this procedure for the model validation.

INTRODUCTION

For a stationary random signal, the autospectral density function (power spectrum) describes its energy distribution with respect to frequency. For a non-stationary random signal, the energy distribution with respect to frequency is time-dependent. Thus, the energy distribution is a function of time and frequency, which can be viewed by a map over the time-frequency plane. There are several methods for decomposing non-stationary signals and generating the energy distribution map [1]. Among these are: (a) the short-time Fourier transform (STFT), which generates a spectrogram; (b) the Wigner-Ville method, creating a time-frequency distribution; and (c) the wavelet analysis, providing a time-scale distribution [2].

Impact signals, such as automobile crash or aircrew ejection responses, have several unique features. They are transient, as an impact event usually takes place in short time duration. The dominant or major component of an impact response, such as the gross motion of a vehicle in a frontal impact, can be considered as deterministic. Some impulses or short-time vibrations, resulting from secondary impact or structure failure, may occur in certain time spans. The impact signals are usually contaminated with random noise. Whereas an impact signal can be broadly considered as a non-stationary signal, a new method for energy distribution analysis is desired in order to describe its unique features.

As a new tool for signal analysis, wavelets are localized in both frequency and time domains, which match the major characteristics of impact responses. Therefore wavelets are used in this paper. Impact responses will be decomposed on orthogonal wavelet bases or wavelet packet bases. Based on these orthogonal decompositions, the energy distribution of an impact signal will be defined and the method for the energy distribution analysis will be developed. The energy distribution analysis will be performed on automobile crash signals from an actual test and from computer simulations. As a practical application, the energy distribution will be used in the validation of finite element automobile crashworthiness modeling.

WAVELET AND WAVELET PACKET DECOMPOSITION

In a wavelet basis, a signal $S(t)$ can be decomposed as [3-5]

$$\begin{aligned}
S &= A_1 + D_1 \\
&= A_2 + D_2 + D_1 \\
&= \cdots \\
&= A_J + \sum_{j \leq J} D_j
\end{aligned} \qquad , \tag{1}$$

where

$$A_j(t) = \sum_k c_{jk}\, \phi_j(t-k), \tag{2}$$

and

$$D_j(t) = \sum_k d_{jk}\, \psi_j(t-k). \tag{3}$$

Here, A_j and D_j are referred to as the approximation and the detail of S at level j; $\phi_j(t)$ and $\psi_j(t)$ are the scaling function and the wavelet function at level j for reconstruction; and c_{jk} and d_{jk}, given by wavelet transforms, are scaling function coefficients and wavelet coefficients at level j and time shift k, respectively. The decomposition of a signal in a wavelet basis is illustrated in Fig. 1(a).

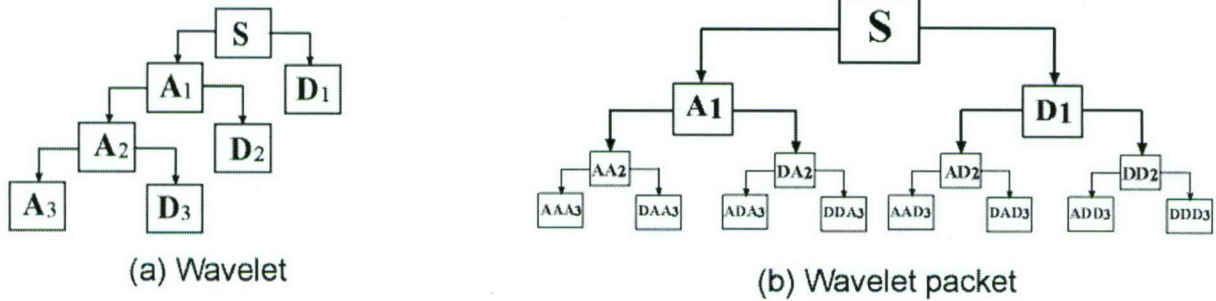(a) Wavelet   (b) Wavelet packet

Fig. 1. Decomposition in a wavelet basis and a wavelet packet basis

The total band is divided unevenly using a wavelet analysis. To obtain the decompositions over evenly spanned sub-bands, the wavelet packet analysis can be used [3,4]. As illustrated in Fig. 1(b), the wavelet packet analysis is a generalization of the wavelet analysis. In the orthogonal wavelet decomposition procedure, successive approximation coefficients are split into two parts, but successive details are never re-analyzed. In the corresponding wavelet packet situation, each detail coefficient vector is also decomposed into two parts using the same approach as in approximation vector splitting. This offers a more complete analysis of a signal.

In a wavelet packet basis, the decomposition of the signal $S(t)$ at level j can be expressed as

$$S(t) = \sum_{n=0}^{2^j - 1} \sum_k q_{jnk}\, w_{jn}(t-k), \tag{4}$$

where $w_{jn}(t)$ are wavelet packet atoms [3,4]; q_{jnk} are wavelet packet coefficients; n is the frequency index which is in accordance with the natural order of the nodes at this level; and k, the time shift, is the position index. For a J-level decomposition, from a complete wavelet packet decomposition tree there are more than $2^{2^{J-1}}$ different ways to encode a signal [6], for which a more general expression is

$$S(t) = \sum_{(j,n)\in I} \sum \sum_{k} q_{jnk} w_{jn}(t-k),$$ (5)

where I is an index set which is chosen as

$$I = \{(j_0, n_0), (j_1, n_1), ...\},$$ (6)

such that intervals $[2^{j_i} n_i, 2^{j_i}(n_i + 1))$ are disjointed and cover the entire interval $[0, \infty)$ [7]:

$$\bigcup_{i=0}^{\infty} [2^{j_i} n_i, 2^{j_i}(n_i + 1)) = [0, \infty).$$ (7)

One such decomposition, which is picked from the decomposition tree of Fig. 1 (b), can be, for instance,

$$S = AA_2 + ADA_3 + DDA_3 + AAD_3 + DAD_3 + DD_2.$$ (8)

As the number of ways for the decomposition of a signal in wavelet packet bases may be very large and since explicit enumeration is generally unmanageable, it is necessary to find the optimal decomposition in terms of certain entropy criteria, computable by an efficient algorithm [3,4]. These criteria include Shannon, Threshold, Norm, and Log Energy, which are available in the MATLAB wavelet toolbox [4].

SIGNAL ENERGY DISTRIBUTIONS

In a wavelet analysis, if orthogonal wavelets are used, all wavelets $\psi_j(t)$ should be orthogonal to the scaling functions $\phi(t-k)$. Furthermore, the wavelets $\psi_j(t-k)$ should be mutually orthogonal, and the scaling functions $\phi(t-k)$ should be mutually orthogonal also [3, 5]. This leads to the following relations [3]:

$$\int_{-\infty}^{\infty} \phi_j(t-m)\phi_j(t-n)dt = \delta(m-n),$$

$$\int_{-\infty}^{\infty} \phi_j(t-m)\psi_j(t-n)dt = \delta(m-n),$$ (9)

$$\int_{-\infty}^{\infty} \psi_j(t-k)\psi_J(t-K)dt = \delta(j-J)\delta(k-K),$$

where δ is the Kronecker delta function. Accordingly, the hierarchical decomposition of Eq. (1) is orthogonal. That is,

- A_J is orthogonal to $D_J, D_{J-1}, D_{J-2},...$,
- D_j is orthogonal to D_k for $j \neq k$.

It follows that in terms of the energy that $S(t)$ contains,

$$\|S\|^2 = \|A_1\|^2 + \|D_1\|^2$$
$$= \|A_2\|^2 + \|D_2\|^2 + \|D_1\|^2$$
$$= \cdots \qquad ; \qquad (10)$$
$$= \|A_J\|^2 + \sum_{j \leq J} \|D_j\|^2$$

Furthermore, from the relations expressed by Eq. (9),

$$\|S\|^2 = \sum_k c_{Jk}^2 + \sum_{j=1}^{J} \sum_k d_{jk}^2 , \qquad (11)$$

which describes the energy distribution of a signal over scales (frequency bands) and time spans.

Wavelet packet functions (atoms) are created by taking linear combinations of the usual wavelet functions [3,4,7]. The wavelet packet bases are a generalization of wavelet bases and inherit properties such as orthogonality and smoothness from their corresponding wavelet functions [7]. Therefore, the relations similar to those described for the wavelet analysis may exist in the wavelet packet analysis. Among these is, for orthonormal wavelet packet atoms,

$$\int_{-\infty}^{\infty} w_{jn}(t-k) w_{JN}(t-l) dt = \delta(j-J)\delta(n-N)\delta(k-l) . \qquad (12)$$

If the decomposition of a signal is performed on orthogonal wavelet packet bases, in terms of the signal energy,

$$\|S\|^2 = \sum_{n=0}^{2^j - 1} \sum_k q_{jnk}^2 , \qquad (13)$$

which corresponds to Eq. (4), and

$$\|S\|^2 = \sum \sum_{(j,n) \in I} \sum_k q_{jnk}^2 , \qquad (14)$$

which follows from Eq. (5).

The signal energy distribution with respect to frequency and time can be determined using Eq. (13) if the signal is decomposed at a certain level and on evenly spanned frequency bands. Likewise, the signal energy distribution with respect to scale, frequency, and time can be analyzed based on Eq. (14) when the signal is decomposed on an optimal tree that may spread over multiple levels and on unevenly spanned frequency bands.

## ENERGY DISTRIBUTION ANALYSIS FOR MODEL VALIDATON

For the finite element (FE) modeling of automobile crashworthiness, the model validation is often conducted in terms of entire vehicle motions and acceleration responses at certain critical points [8,9]. When acceleration responses are compared in the validation, the time histories measured in actual tests and obtained from simulations are often plotted curve by curve. The agreement or discrepancy between the test data and simulation results can be evaluated by visual inspection of the amplitudes, phases or timing of peaks, and pulse shapes of both signals. While visual inspection is intuitive, direct, and easy, it can be qualitative, subjective, and coarse. Quantitative and objective evaluations based on statistical or other scientific analyses can be more accurate and meaningful, and thus are desirable.

The energy distribution describes the variation of amplitude with respect to frequency and time. The comparison of the energy distributions of two signals provides a description of the agreement and discrepancy between them; thus it can be used for the validation of a computational model where the simulated responses using the model are compared with actual responses from tests. When a pair of signals from test and simulation are decomposed in the same wavelet packet basis with the same entropy criterion, the best decomposition trees for them may be different. This means that the signal energy distributions based on respective best decomposition trees may not be comparable. Therefore, both signals need to be decomposed on the same tree, in order for their energy distributions to be comparable. As a choice, both signals can be decomposed at the same level in a wavelet basis (Eq. (1)) or a wavelet packet basis (Eq. (4)).

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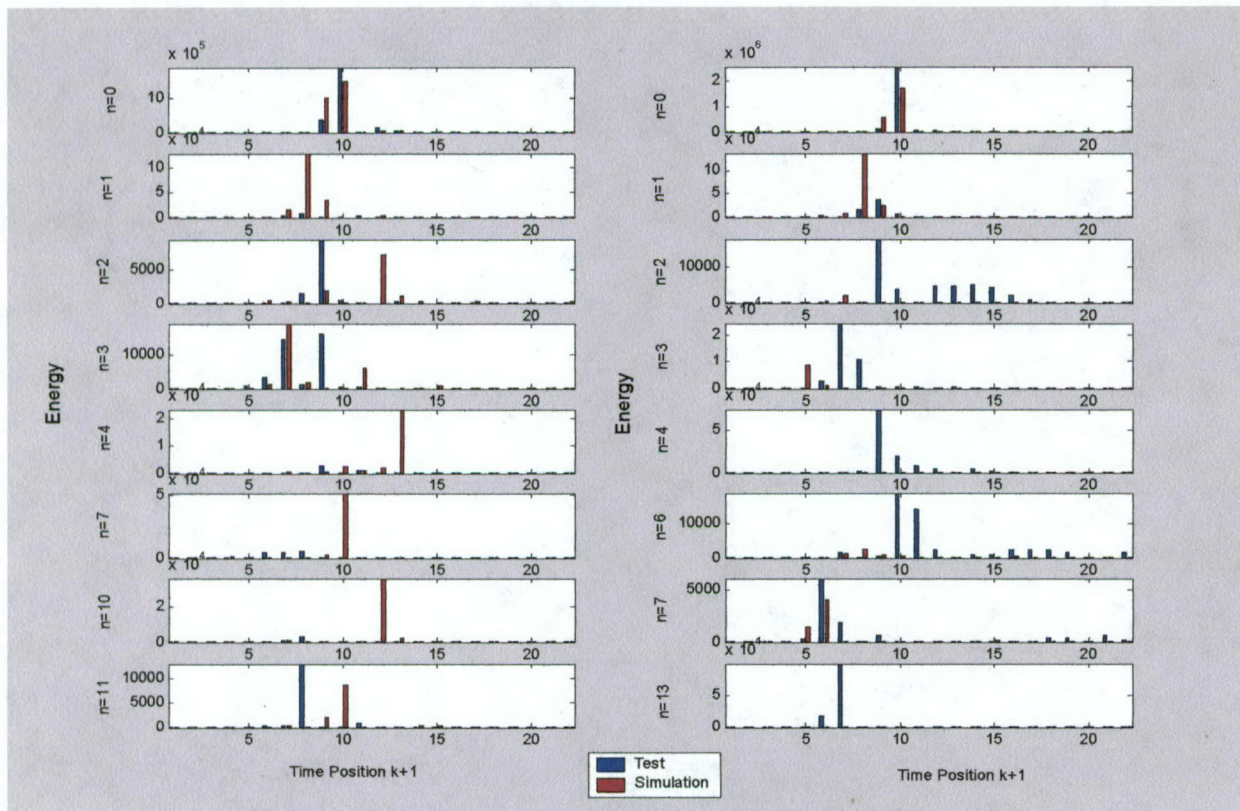
(a) Engine bottom   (b) Engine top
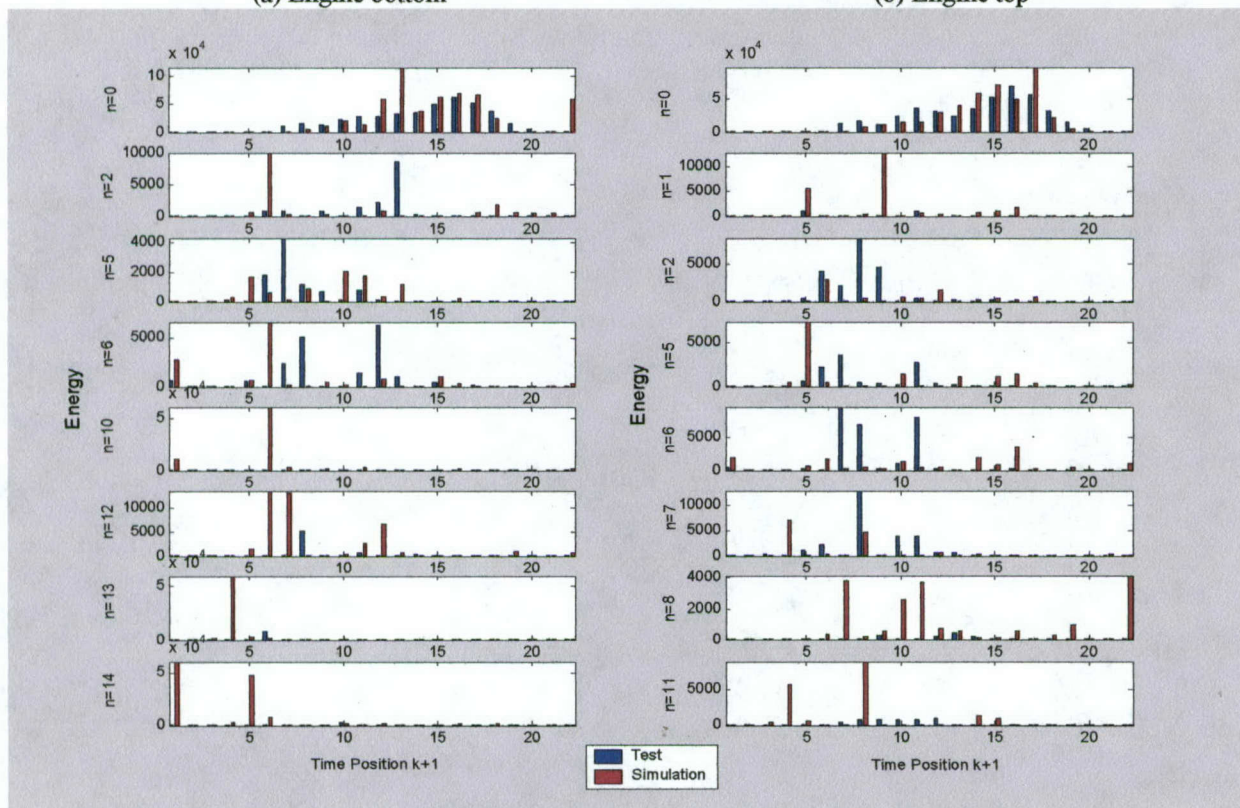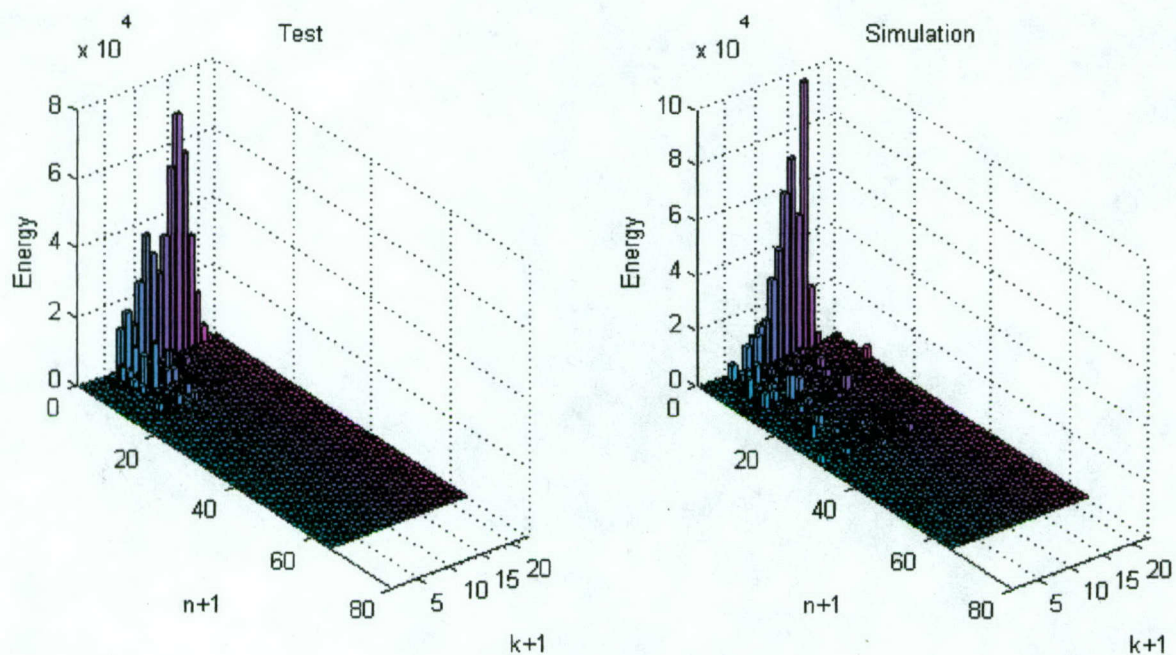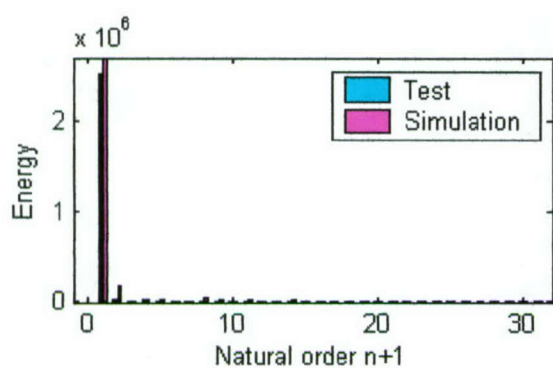
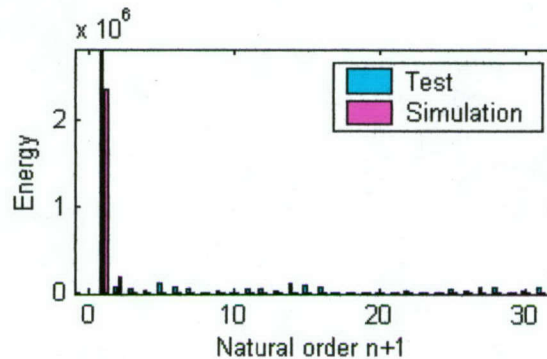(c) Left-rear cross member   (d) Right-rear cross member
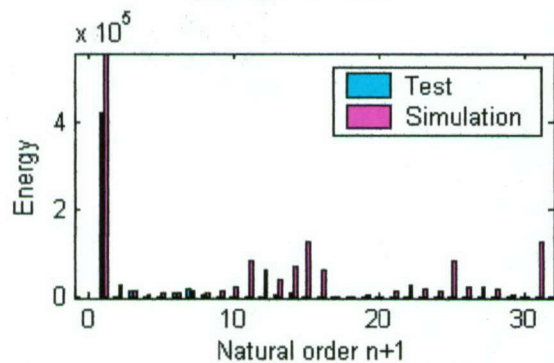
Fig. 6. Energy distributions with respect to k

(d) Right-rear cross member
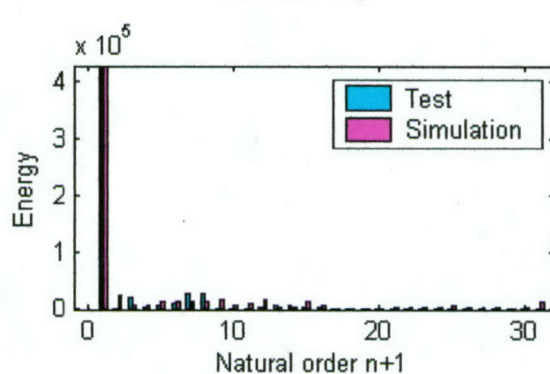Fig. 4. Energy distribution with respect to n and k

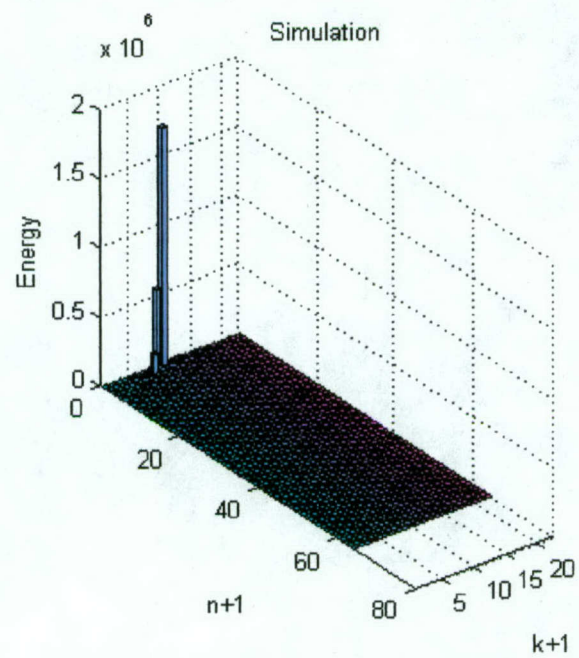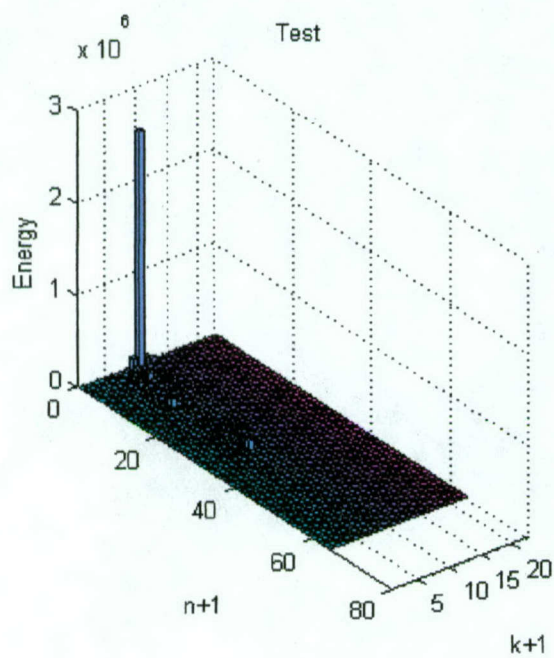

(a) Engine bottom

(b) Engine top

(c) Left-rear cross member
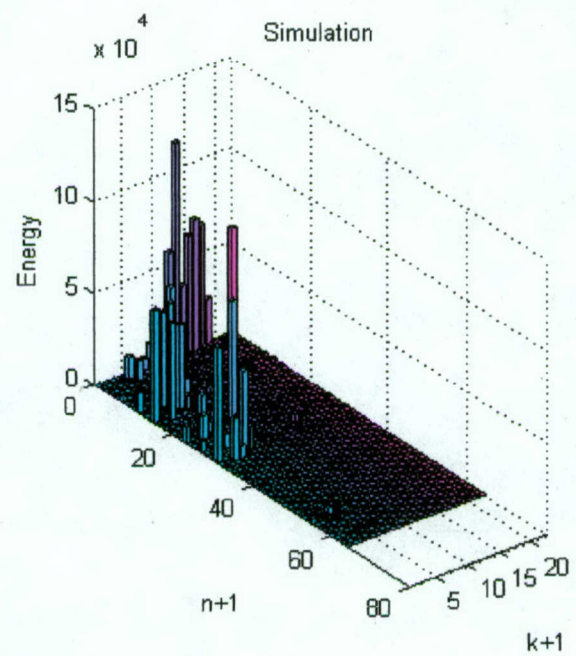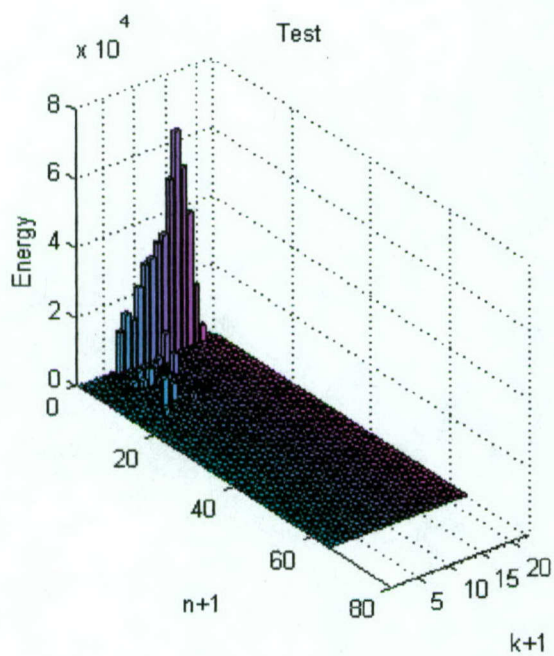

(d) Right-rear cross member
Fig. 5. Energy distributions with respect to n

(b) Engine top

(c) Left-rear cross member

Energy Distribution Analysis and Model Validation

The energy distributions of these signals with respect to the natural order and time location, which are calculated using Eq. (16), are displayed in Fig. 4. This figure provides a three-dimensional view of the signal energy distributions. For the acceleration responses at the locations of engine bottom (Fig. 4(a)) and engine top (Fig. 4(b)), the energies of both signals from test and simulation concentrate on the first node with natural order $n=0$ corresponding to the lowest frequency band, and spread over the time span from the position $k=5$ to $k=15$. For the acceleration responses at the locations of the left- and right-rear cross member (Fig. 4(c) and (d)), both signal energies are primarily contained at the first node, but there are significant amounts of energy distributed over high order nodes, especially for the response at the left-rear cross member from the simulation. The energies at these nodes spread over a wide time span.

To find the energy distributions at each node, the energy distributions with respect to the natural order are calculated using Eq. (15) and are illustrated in Fig. 5. Since the energy distributions over high order nodes ($n \geq 32$) are insignificant, as shown in Fig. 4, the energy distributions in Fig. 5 are displayed only for the natural order up to $n=31$. The entire range is from $n=0$ to $n=63$. In addition to providing a more clear view of the energy distributions at each node as compared to Fig. 4, Fig. 5 offers the comparison between the signals from test and simulation. As far as the principal component of energy that is placed on the first node is concerned, the agreement between the test data and simulation results using the FE model is sound for the acceleration response at the right-rear cross member, good at the engine bottom, fairly good at the engine top, and still close at the left-rear cross member where considerable amounts of energy are contained at high order nodes (Fig. 5 (c)).

The energy contained in each node can be further analyzed from the perspective of its distributions over time, as shown in Fig 4. To obtain a more intuitive and clear view, based on Eq. (16), two-dimensional graphs are derived from Fig. 4 for the nodes that have major signal energy distributions. These graphs are shown in Fig. 6. As far as the energy contained in the first node is concerned, the simulation with the FE model has attained a sound agreement with the actual test in terms of the amount of energy in this frequency band, but the distributions over time spans are not quite agreeable. For other higher-order nodes, these distributions are basically not agreeable between the simulation and the test.
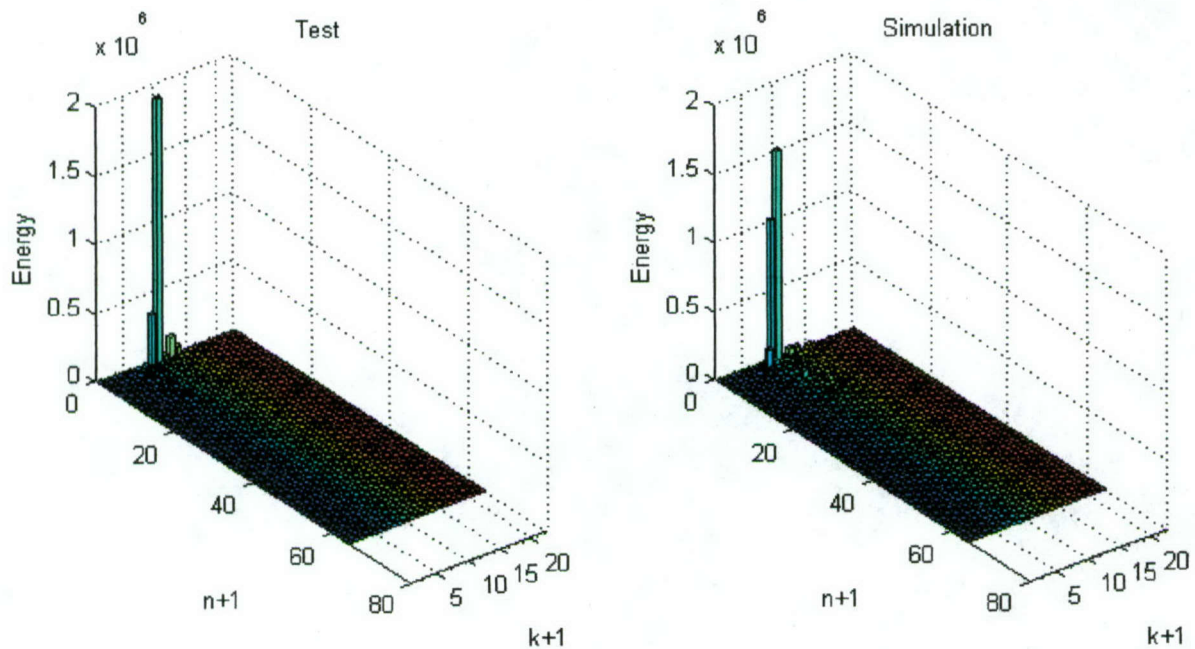
(a) Engine bottom

CONCLUDING REMARKS

Based on the decomposition of a signal on wavelet packet bases, two types of signal energy distributions have been established: (a) the energy distributions with respect to the natural order of nodes or the frequency index n; (b) the energy distributions with respect to the time position k and the frequency index n. The unique characteristics of a wavelet analysis render the energy distributions based on wavelet or wavelet packet decompositions to be especially suitable for the analysis of impact signals, such as automobile crash responses. The gross motion component of an impact signal is readily represented by the approximation. The short-time impulses and vibrations localized in both time and scale (frequency) can be expressed easily by the details. If necessary, de-noising and compression, two powerful operations provided by a wavelet analysis, can be employed to eliminate noise and insignificant components, so that the energy distribution can focus on significant components.

It is important to realize that the energy distributions over the time-frequency plane based on wavelet or wavelet packet decompositions may look quite different when different wavelets are used. Unfortunately, this problem occurs in other methods (such as STFT) also [2]. Therefore, caution needs to be taken when a wavelet time-frequency spectrogram is interpreted. The characteristics of the wavelets used in the decomposition need to be taken into consideration also.

Due to transient and non-stationary characteristics of automobile crash responses, validation methods and criteria based on conventional statistic analysis, correlation analysis, or spectral analysis used for stationary signals may be neither appropriate nor efficient. Comparing energy distributions between a pair of signals from tests and simulations provides a way to evaluate simulation results and to validate the model from certain perspectives. The distributions of type-b provide more detail and complete information of a signal than the distributions of type-a. Accordingly, the validation based on the energy distributions of type-b is more comprehensive and rigorous than the validation based on the distributions of type-a.

Whereas the energy distributions defined above provide a common basis for the comparison of two signals, they are mainly concerned with the amplitudes of signals. It is sometimes necessary to compare the shapes of pulses and the timing of peaks between two responses. This problem can be treated from the perspective of the correlative relationship between two signals [14,15].

The validation of a 1997 Honda Accord FE model for full frontal impact has been performed in this paper. The impact responses from a simulation and an actual test have been evaluated and compared. The results show that, from the perspectives of the energy distributions over frequency bands, the agreement between the simulation results using this FE model and the test data is good for the gross motions at the locations of the engine bottom, engine top, and right-rear cross member, and fairly good for the gross motion at the location of the left-rear cross member.

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